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Complex dynamics in soil temperature: A case study of Southern Nigeria

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Abstract
In this paper, the soil temperature dynamics in south-western Nigeria is investigated using nonlinear time series analysis approach. The 5-mins interval soil temperature data observed continuously over a period of 1 year (from January to December, 2012) were analysed to investigate the existence of chaos. We employed the method of false nearest neighbour carefully interlaced with the average mutual information method to reconstruct the original attractor of the soil temperature's evolution process. The Lyapunov exponent, in addition to the strange attractor of the process, was also employed to identify chaos in the process. Numerical simulations were performed to show the validity of our approach. Tabular and graphical are presented. The results of the computation show the existence of chaos in the soil temperature.

Keywords: Soil temperature; chaos; nonlinear time series; false nearest neighbour; average mutual information

1. Introduction

In recent times analysis involving the measurement of soil temperature has been on the increase. This is due to the fact that the knowledge of soil temperature is valuable for many reasons. Soil temperature, among others is of great importance in agricultural processes as it affects the soil's physical, chemical and biological processes (Neitsch et al, 2005; Sandor and Fodor, 2012; Kunkel, et al 2016). For example, the germination of seedlings, the growth of plant and the uptake of nutrients are dependent, among other metreological factors, on the soil temperature. Excessively high soil temperature results in extreme condition known as drought, which causes environmental and agricultural catastrophe. Flood is another undesirable environmental condition which stems from the occurrence of extremely low soil temperature. Furthermore, soil temperature is important in solar energy-based heating and cooling applications, ground based heat pump applications as well as frost forecasting (Mihalakakou, 2002; Shah et al, 2019; Sun et al, 2020). All these are quite controllable if the dynamics of this temperature is known prior to their occurrence. Similarly, a good knowledge of soil temperature is particularly useful in agricultural metreology. In addition to this, It is well known that the quantity of solar radiation incident on a particular area of land affects soil temperature (Russell 1973, Haskell et al, 2010) and this quantity of solar radiation also depends on such parameters as the aspect, slope, percent canopy cover (Haskell et al, 2010; Sandor and Fodor, 2012) etc. It is not surprising therefore, that different subcontinents across the globe are predisposed to varying soil temperature dynamics. Besides, soil temperature is a pivotal variable in the global discuss to better place and monitor the global warming theory (Stocker et al, 2013).

Soil temperature measurement is often required for significant intervention in real world processes and to forestall extreme conditions, such as flood and draught including global warming. Unfortunately, actual measurement at the time of application is practically impossible because of the spontaneous nature of such extreme temperature effects, hence the need for the characterisation of its overall dynamics, in order to understand the underlying behaviour of this dynamics. Nonlinear time series analysis provides very useful approach in this regard, which helps us to understand not only the empirical data as observed but also the complicated details of the nonlinear behaviour of the process.

In all the studies above, the existence of chaos has not been established. However, the proportional course-effect mechanism is not applicable in many real-world situations. In fact, chaotic behaviour has been found in temperature related dynamics. For instance the source of motivation for this work is the results of the study by Fenga et al, 2019, in which the presence of chaos in the near-surface temperature was investigated. The present analysis includes the estimation of the embedding dimension and the Lyapunov exponent.

Time series analysis has paved the way for numerous developmental and interdisciplinary applications in increasing fields of studies, which include medical sciences, natural sciences and agricultural sciences. Examples of these applications are ubiquitous, among which is the characterisation of the dynamics exhibited by empirical data such as rainfall-runoff dynamics (Sivakumar et al, 2001) and the ionospheric plasma dynamics (Ogunsua et al, 2014), as well as the prediction of future dynamical behaviours of the observed data ranging from the forecast of future trend of climatic and metreological coordinates (Günes et al, 2014) to those in finance. The interest in the analysis of these real-world observations along temporal dimension otherwise called time series, originates from the fact that such data are available in large quantity taken over long periods, which fascinatingly display the previous dynamics of the measured variable. Some of these dynamics possess obvious trends, while some others possess a rather complex, but deterministic evolutions. However, understanding of the actual nature or classification (whether trendy, cyclical, seasonal, or complex) of time series of interest is fundamental to further and future applications. Therefore the focus of this present work is to characterise surface soil temperature time series and hence its dynamical behaviour (Liu, 2010; Athiyarath et al, 2020).

The presence of chaos in dynamical systems is not undesirable all the time. This phenomenon does enhance determinism sometimes, while presenting chaoticity. Hydrological variables such as rainfall and run-off have been characterised and predicted with better precision by utilising the chaoticity in their time series (Puente and Obregon, 1996; Sivakumar, 1999; Stehlic, 1999; Kasovskaia et al, 1999; Sivakumar et al, 2001). In addition, chaos is a valuable phenomenon in the functionalities of human brain (Liu, 2010)

Over the years, considerable effort has been committed to unravel the methodologies and concepts that help to understand the dynamical behaviours of time series observed in climatology including temperature dynamics. In this regard, remarkable success has been reported in the literature (e.g. Mihailovi? and Mimi?, 2012; Ogunlela, 2003; Sandor and Fodor, 2012; Nwankwo and Ogagarue, 2012). However, majority of these works considered the dynamics of soil temperature as periodic; meanwhile the model attempt that uses the force-restore approach to calculate the ground interface temperatures as well as other climatic variables reported that soil temperature exhibits complex behaviours (Mihailovic and Mimic, 2012). This behaviour is due to a lot of complex interactions between essential processes that take place at the boundary e.g. chemical, biological and physical processes including the transformation of energy at the boundary. Indeed, the application of chaos theory to characterise time series data have been proven to be more effective in a number of areas including hydrology (Sivakumar et al, 2001; Dhanya and Kumar, 2011). Nonlinear time series may be reproduced (with respect to prediction) with minimal error using deterministic chaos theory without recourse to the underlying mechanisms. The advantage of deterministic chaos theory over the statistical approach, which employed few statistical parameters to analyse the dynamics (Sivakumar et al, 2001), is that the whole process that engendered the observed behaviour of the time series is fully captured. In order to typify nonlinear time series such as the one in this present work, sufficient chaos identifiers have been proposed and widely used in the literature (Ott, 1993; Alligood et al, 1997; Liu, 2010). Notable among them are the delay embedding procedure by Takens, 1981, which carefully incorporates the estimation of the hitherto unknown system's coordinates (dimensions) to unfold the attractor of the time series in the phase space and the calculation of the Lyapunov exponent, which has widely been employed as a confirmatory estimator of chaotic signature. These parameters are sufficient because they are invariants of the system (Liu, 2010).

2. Data used and methods of analysis

The time series of the soil temperature (ST) data observed with 5-minutes interval in Redeemer's university, located in southern Nigeria, is analysed to examine and characterise the dynamics of ST in Nigeria. The ST data were measured using Yong's temperature sensor, calibrated to capture temperatures in the range -50 to 50 oC within oC accuracy. This present study utilised 105,408 data points taken over 12 months - from 12:00 am January 1st to 23:55 pm December 31st, 2012. The climate of the study area is mainly the tropical rainforest. Fig. 1 below shows the position of the study area in Ogun state of Nigeria.

Interestingly, inferences from time series data hinge largely on the successful reconstruction of the unknown phase space of the series (Packard et al, (1980), Kocak et al, 2004). This is necessary to deduce the embedding parameters such as the embedding dimension, *d* and the optimum time delay, *r*. The embedding dimension measures the optimum number of independent variables required to project the attractor of the series unto the phase plane. Many methods have been proposed for the calculation of the embedding dimension, among which is the prominent duo: the Grassberger's correlation dimension (Grassberger and Procaccia, (1981)) and the notion of nearest neighbour distance proposed by Badii and Politi (Bidii and Boliti, 1985), which became well adopted following the literature report by Kennel et al, 1992. A finite embedding dimension is an indication that the reconstructed phase space is non-periodic.

The reconstruction of the phase space is such that a scalar time series X_i, where $i=1,2,,,,N$, is represented by m- dimensional phase space (m is the embedding dimension of the attractor), which is the representation of all the dynamical variables of the system. In this way the attractor gives a rich dynamical insight as regards the complexity of the original dataset. According to Takens (1981); Pachard (1980); Dhanya and Kumar (2010), the reconstructed phase space may be written as

$$Y_j = X_j, X_{j+\tau}, X_{j+2\tau}, \ldots, X_{j+(m-1)\tau} \quad (1)$$

where, $j = 1,2,\ldots,N-(m-1)\tau/\Delta t$ d is the dimension of the attractor and Δt is the time interval between observations. The embedding dimension is chosen such that $m \geq d$. The time delay plays a crucial role in the desirability of the reconstructed attractor. Supposing the time delay τ^* is chosen, where, $\tau^* < \tau$ there exists a high correlation between adjacent coordinates on the phase space. The implication of this phenomenon is that the coordinates become inseparable leading to topological deformation of the attractor. Another possible scenario is that the delay time is chosen such that $\tau^* > \tau$, In this case, there is significant loss of correlation between adjacent coordinates leading to complete independence between the state variables (Kostelich and Swinney, 1989). Two methods have been widely reported in the literature for choosing the optimum time delay: the autocorrelation method (Dondurur, 2018) and the mutual information method (Fraser and Swinney, 1986). It has been shown that the mutual information function is preferably applied for nonlinear problems (Kocak *et al*, 2004), autocorrelation function measures the linear relationship between neighbours. The method of mutual information is used in this work to estimate τ.

The signature of chaotic behaviour is identifiable via the estimation of the Liapunov exponent corresponding to the chaotic system. This invariant of the system measures the mean exponential rates of divergence or convergence close manifolds in the phase space of the system (Wolf *et al*, 1985). A positive maximum Lyapunov exponent shows the presense of chaos in the data set. For a zero maximum Lyapunov exponent, a limit cycle or a quasiperiodic orbit is implied, while a negative maximum Lyapunov exponent represents a fixed point. The algorithm for the estimation of this exponent often suffers some disadvantages which include partial usage of the dataset, variations in the estimated embedding parameters and noise that may be inherent in the observed dataset (Wolf *et al*, 1985; Sato *et al*, 1987). Here an improved algorithm is considered which is robust and stable against slight changes in data size, values of embedding parameters etc (Kantz, 1994; Rosentein *et al*, 1993). After Rosentein *et al*, (1993), the largest Lyapunov exponent is calculated from the lines given by the equation

$$y(i) = \frac{1}{\Delta t}\left\langle \log d_j(i) \right\rangle \tag{2}$$

where $d_j(i)$ represents the distance between *jth* pair of nearest neighbours after $i\Delta t$ seconds, and *(.)* is the average over all *j*. The application of least-squares fit to the average lines given by Eq. (2) provides the largest Lyapunov exponent λ_l.

Table 1. The statistical parameters of the soil temperature data.

Parameters	Min	Max	Mean	Med	Mode	Std	Range
Values	24.01	35.83	28.81	28.73	28.84	1.75	11.82

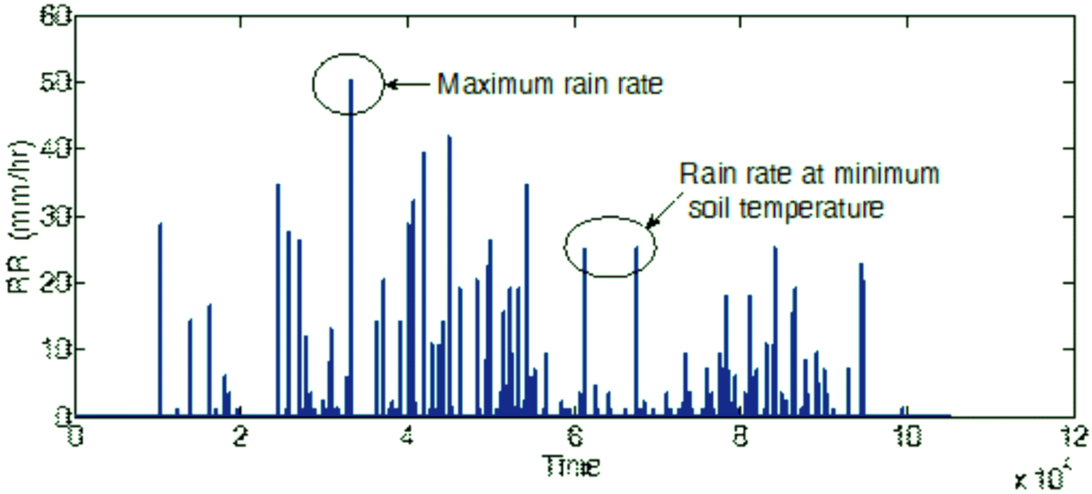


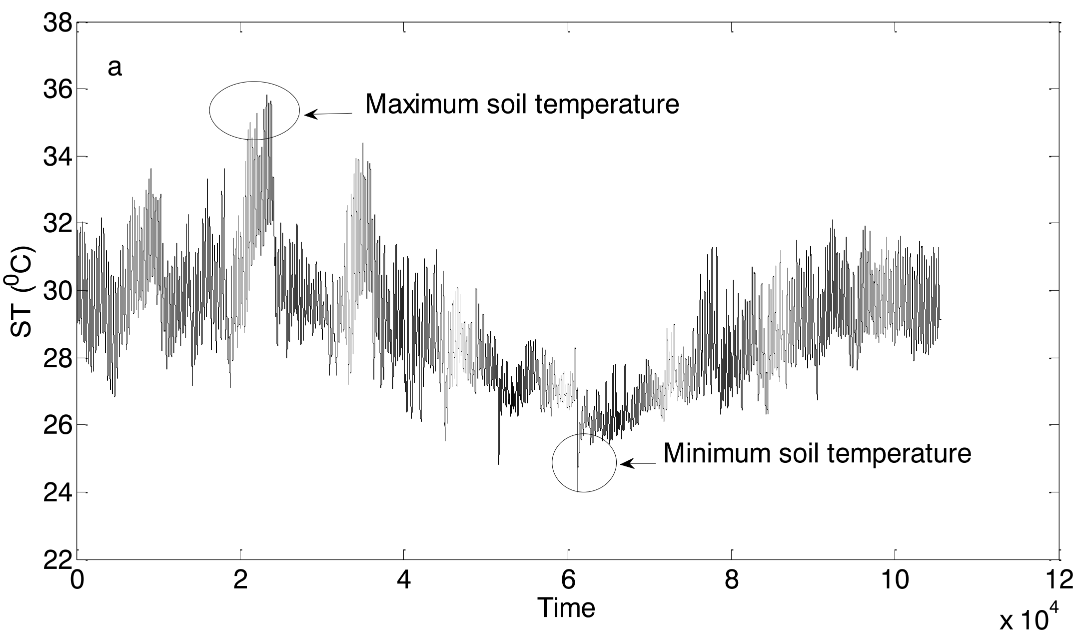
Fig. 1: The time series of the rain rate data for the year 2012.

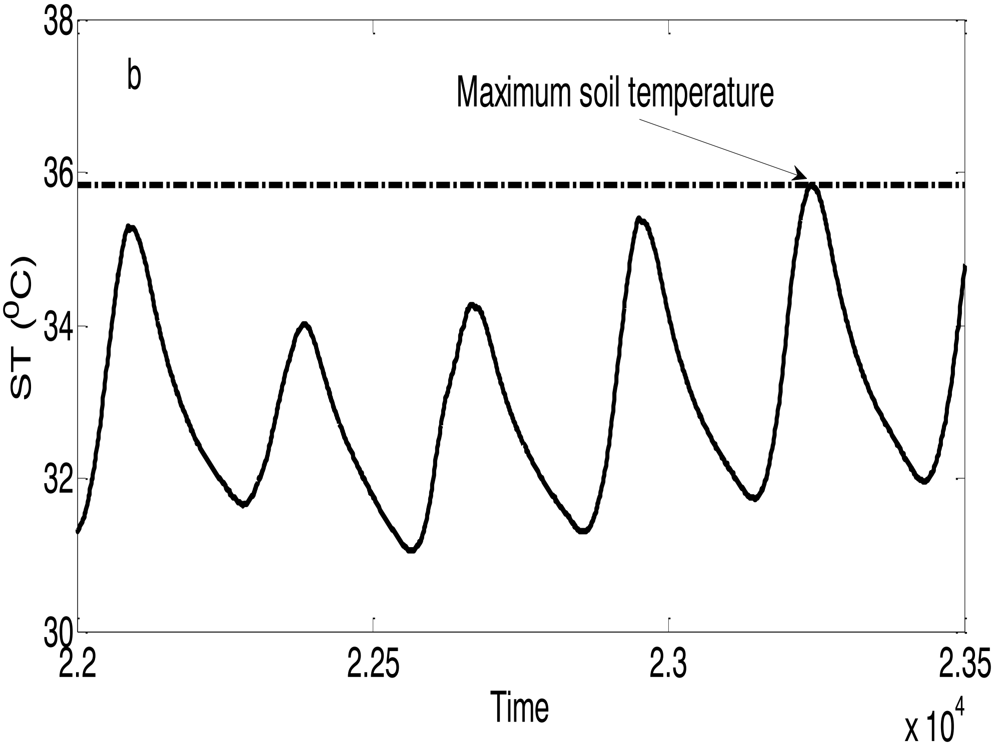
3. Results and discussion

The soil temperature series spanned a period of 12 months as highlighted in section 2, from 1st January to 31st December, 2012 (Fig. 3a). Before the generic report of the dynamical behaviour of the ST time series, it is interesting to note some descriptive parameters that let us understand some short-term or local behaviour of the ST. In this way, the non-stationary nature of the series would be appreciated, especially as regards the importance of ST in agriculture. Agricultural practice accord regards to seasons such as rainy and dry seasons, during which agricultural activities are planned accordingly (this might be relatively suppressed by irrigation system though).

Consequently, we employed descriptive statistical tools to this seasonal trend. Tab.1 gives the summary of the statistical features of the time series. The average annual ST is 28.81 0C, with the deviation of 1.75 0C. The difference between the extreme values of the time series in the year 2012 stands at 11.82 0C, with 28.84 0C being the most observed value. Table 1 also showed that the annual ST variability attained a high of about 35.83 0C at the Redeemer's University during the dry season, especially at its peak in March (please see Fig. 2b). The dry season in Nigeria is the period between November and March. It is also shown in Table 1 that the soil temperature has annual lows in the neighbourhood of 24.01 0C during the rainy season, notably between May and August.

Fig. 1 showed the time series of the rain rate data observed at the same time interval with the soil temperature data in the same station. Apart from the precipitation, soil temperature is also influenced by properties of the soil on which the process of heat flow in the soil depends; the vegetation (Anctil et al, 2008; Zheng et al, 1993; Kang et al, 2008) as well as the possible atmospheric irregularities (e.g. Ogunsua et al, 2014; Hu and Feng, 2003). Fig. 2a gives the ST variability in the year under study. Figs. 2b and 2c showed the expanded view of the ST variability. Fig. 2c showed that the lowest temperature was observed on the last day of July (31st July, 2012). This plot showed a sharp decline in the ST to the minimum ST point. The observation of the low ST may be attributed mainly to the consequence of a long period of soil cooling activity due to the relatively heavy rainfall, which lasted for almost the whole day with the highest rain rate of 25.2 mm as shown in Fig. 1. It is necessary to note here that the lowest observed ST does not necessarily correspond to the occurrence of the highest rain rate, which was observed in April with the rain rate of 50.4 0C as shown in Fig. 1.




b
Maximum soil temperature
ST (°C)
38
36
34
32
30
2.2
2.25
2.3
2.35
Time
x 10^4

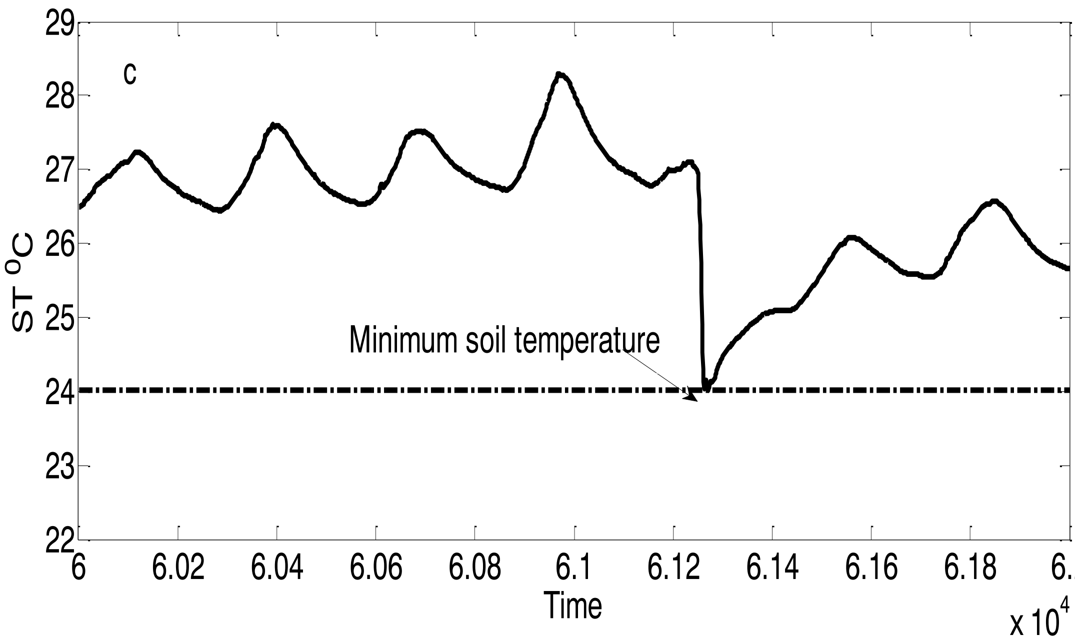

c
Minimum soil temperature
ST °C
29
28
27
26
25
24
23
22
6
6.02
6.04
6.06
6.08
6.1
6.12
6.14
6.16
6.18
6.2
Time
x 10^4

Fig. 3: Soil temperature time plots, (a) for the 12 months data, (b) for a shorter period to show the peak temperature and (c) for a shorter period to show the lowest temperature.

Very critical in the process of finding the topological equivalence of the ST dynamics attractor is the way the delay time and embedding dimension are chosen (Takens, 1981). The idea of applying time delay to derive new d-dimensional series of vectors from the original attractor of the system is to track as much as possible information from the collection of observations, which is comparable with the slow and fast scales formalism otherwise called the multiple time scales method applicable in perturbation analysis. While the fast time scale could be too fast as to have left some information necessary to recover the attractor of the dynamics underlying the behaviour of the observed variable, a slow time scale such as proposed in the delay embedding procedure includes such information in the reconstructed vector series. Therefore, a carefully chosen delay time helps obtain a sufficient and appropriate form of Eq. (1) for further analysis. To this end, based on the discussion in section 2 we applied the average mutual information technique to obtain the delay time for the ST time series. Fig. 3(a) shows the average mutual information plot, with its first minimum corresponding to point 2 on the delay time axis; hence giving the optimal delay time, 2=t. In Fig. 3(b), the embedding dimension is calculated using the fraction of false neighbours (FNN). The value of the embedding dimension at which FNN drops to zero is chosen as the optimal value, i.e. $6=d$. The implication of this is that six coordinates are necessary to correctly display the attractor of the ST dynamics in the phase space. The strange attractor corresponding to this dynamics is shown in Fig. 3(c). The attractor shows that the behaviour of the ST dynamics is sensitive to initial condition.

The chaoticity or otherwise of any dynamical system can be identified by the value of the Lyapunov exponent, as earlier explained in section 2. Fig, 4 shows the variation of the Lyapunov exponent with time for the ST dynamics. The calculated maximum Lyapunov exponent is 0.0061. This identifies the dynamics as chaotic, which also serves as a confirmatory measure for the chaotic nature of the ST dynamics.

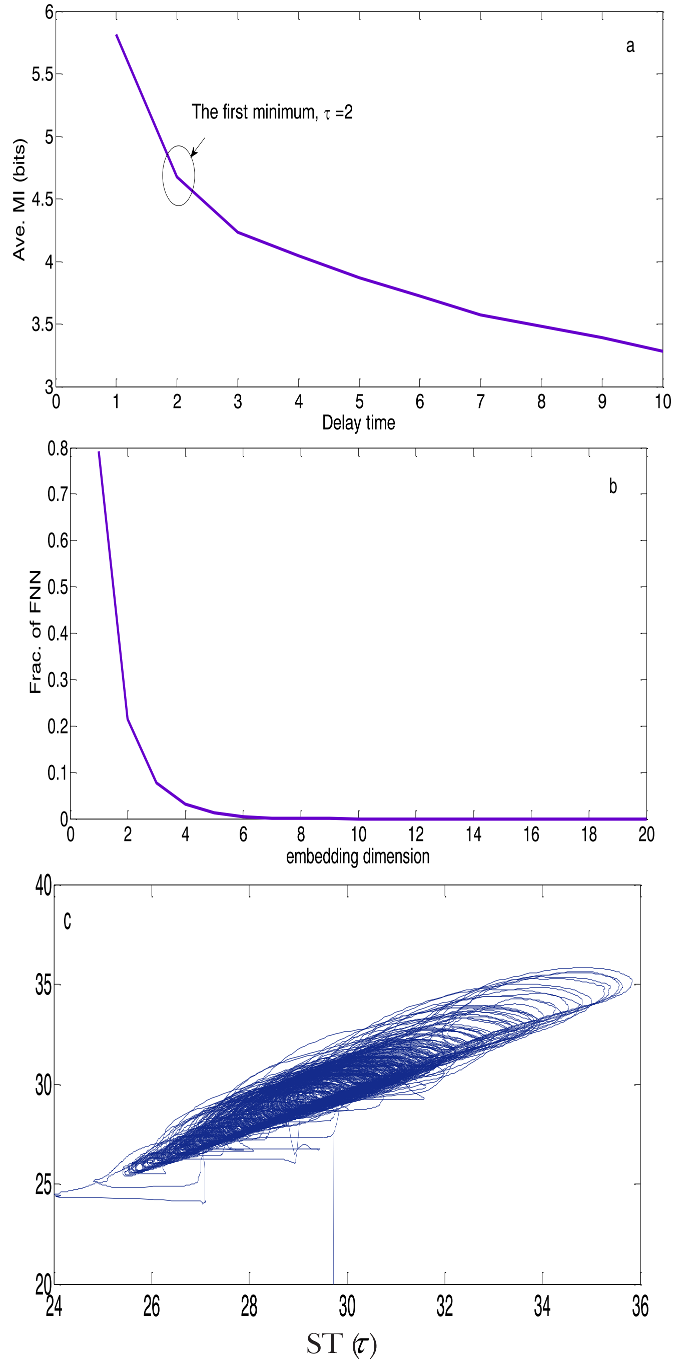


Fig. 3: The delay embedding plots: (a) the average mutual information (Ave. MI) against delay time (b) the fraction of false nearest neighbour against embedding dimension (c) the reconstructed phase space of the soil temperature dynamics, with $\tau = 2$

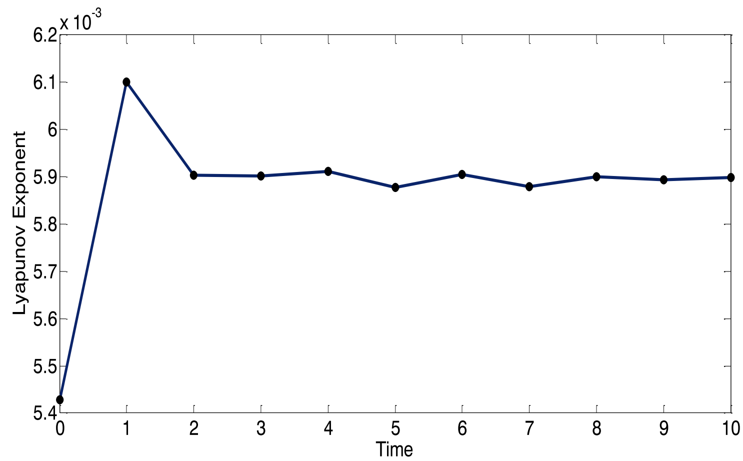


Fig. 4: The Lyapunov exponent of the soil temperature dynamics

4. Conclusions

In this present work, we aimed at the possibility of understanding the behaviour of soil temperature dynamics from the perspective of nonlinear time series analysis. The 5-mins interval soil temperature data observed continuously over a period of 1 year (from January to December, 2012) were analysed to investigate the existence of chaos. We employed the method of false nearest neighbour carefully interlaced with the average mutual information method to reconstruct the original attractor of the soil temperature's evolution process. The Lyapunov exponent, in addition to the strange attractor of the process, was also employed to identify chaos.

The results indicated a finite value for the embedding dimension for the system, indicating that the system is chaotic. Furthermore, the positive value of the Lyapunov exponent is a confirmatory result as regards the chaotic napture of the soil temperature time series. In general, this present study suggested that it is possible to understand the underlying process of the soil temperature evolution via the application of nonlinear dynamics theory.

This present work addressed a significant aspect of our proposed study on the soil temperature time series. The understanding of the chaotic nature of the time series gives the opportunity for further investigation such as the prediction of soil temperature, a basic necessity for agricultural processes as well as the control of extreme temperature conditions.

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